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Complete integrability of N-coupled higher-order nonlinear Schrödinger equations in nonlinear optics

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Abstract. The coupled higher-order nonlinear Schrödinger equations are generalized to an N-coupled system which governs the propagation of N fields in a fibre with all the higher-order effects like third-order dispersion, Kerr dispersion and stimulated Raman scattering. The explicit Lax pair is constructed and the soliton solutions are obtained using the Darboux–Bäcklund transformation. Finally, the Hirota bilinear form for the N-coupled system equations is also presented.

It has been realized that the concept of solitons in optical communication has grown in stature and attained a status such that all future communications will be of soliton-type. In optical pulse propagation through a nonlinear fibre, it has been proved that soliton-type propagation is possible by means of a counterbalance of the major constraints in the optical fibre, namely, the group velocity dispersion (GVD) and the self-phase modulation (SPM). SPM is the dominant nonlinear effect in silica fibres due to a Kerr effect arising out of the anharmonic motion of bounded electrons. This type of soliton pulse propagation is governed by the well known nonlinear Schrödinger (NLS) equation derived from the basic wave propagation equations by Hasegawa and Tappert [1]. After the invention of high-intensity lasers, their results were confirmed experimentally by Mollenauer *et al* [2].

For transmitting ultrashort pulses (USPs), it is necessary to consider higher-order effects such as third-order dispersion (TOD), Kerr dispersion and stimulated Raman scattering (SRS) since USPs suffer from these effects, as experimentally reported by Mitschke and Mollenauer [3]. The Kerr dispersion, also known as self-steepening, is caused by the intensity dependence of the group velocity which results in asymmetrical spectral broadening of the pulse since the peak of the pulse travels slower than the wings. The SRS causes a self-frequency shift which is a self-induced red shift in the pulse spectrum as the low-frequency components of the pulse obtain Raman gain at the expense of the high-frequency components. With the inclusion of all these effects, Kodama and Hasegawa [4] have proposed that the dynamics of femtosecond pulse propagation be governed by the higher-order NLS (HNLS) equation. The HNLS equation allows soliton-type propagation only for certain choices of parameters [5].

Wavelength division multiplexing (WDM) using solitons is necessary to propagate more channels simultaneously and also to increase the transmission capacity of the communication system where at least two optical fields are to be transmitted and the system is governed by a coupled NLS equation [6]. A coupled NLS equation was proposed by Manakov, derived

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by taking into account the fact that the total field comprises two fields with left and right polarizations [7]. The coupled equations take the form

$$\begin{aligned} &\mathrm{i}q_{1Z} + c_1 q_{1TT} + (\alpha|q_1|^2 + \beta|q_2|^2)q_1 = 0 \\ &\mathrm{i}q_{2Z} + c_2 q_{2TT} + (\beta|q_1|^2 + \gamma|q_2|^2)q_2 = 0. \end{aligned} \tag{1}$$

The above equations are integrable only for the following parametric choices: (i) $c_1 = c_2$, $\alpha = \beta = \gamma$ and $c_1 = -c_2$, $\alpha = \beta = \gamma$. In a similar manner, for USPs, coupled HNLS (CHNLS) equations have been proposed by us and we have shown that the system is integrable for a particular choice of parameters using the Painlevé singularity structure analysis [8]. The general forms of CHNLS equations are

$$\begin{aligned} &\mathrm{i}q_{1Z} + \alpha_1 q_{1TT} + \alpha_2(|q_1|^2 + |q_2|^2)q_1 + \mathrm{i}\varepsilon[\alpha_3 q_{1TTT} + \alpha_4(|q_1|^2 + |q_2|^2)q_{1T} \\ &\qquad + \alpha_5 q_1(|q_1|^2 + |q_2|^2)_T] = 0 \\ &\mathrm{i}q_{2Z} + \alpha_1 q_{2TT} + \alpha_2(|q_1|^2 + |q_2|^2)q_2 + \mathrm{i}\varepsilon[\alpha_3 q_{2TTT} + \alpha_4(|q_1|^2 + |q_2|^2)q_{2T} \\ &\qquad + \alpha_5 q_2(|q_1|^2 + |q_2|^2)_T] = 0. \end{aligned} \tag{2}$$

As such, the above equations are not integrable but, by using Painlevé analysis, we found that the above system is integrable for certain parametric conditions (discussed in equation (3)). If some restrictions are imposed on the parametric values, one can obtain several integrable, soliton-possessing NLS-type equations: (i) $\varepsilon = 0$, NLS; (ii) $\alpha_3 : \alpha_4 : \alpha_5 = 0 : 1 : 1$, derivative NLS [9]; (iii) $\alpha_3 : \alpha_4 : \alpha_5 = 0 : 1 : 0$, derivative mixed NLS [9]; (iv) $\alpha_3 : \alpha_4 : \alpha_5 = 1 : 6 : 0$, the Hirota equation [10] and (v) $\alpha_3 : \alpha_4 : \alpha_5 = 1 : 6 : 3$, the Sasa–Satsuma case [11]. Hence, with the inclusion of all the higher-order terms, the CHNLS equations are found to be integrable only for the following choice of parameters: $\alpha_1 = \frac{1}{2}$; $\alpha_2 = 1$; $\alpha_3 = 1$; $\alpha_4 = 6$ and $\alpha_5 = 3$. If we put the condition $q_2 = 0$, equation (2) reduces to the completely integrable HNLS equation. With the inclusion of TOD and Kerr dispersion, Tasgal and Potasek [12] have studied the soliton aspects. In a recent paper, as in the case of a CNLS system, the next hierarchy of CHNLS equations have been reported for the first time [13]. In our earlier work, we have shown that the system of two-coupled HNLS equations can be generalized to the integrable form of N-coupled equations and proposed the possibility of the existence of solitons in them [14]. As an extension of that work, we establish the complete integrability properties of the system of N-coupled HNLS equations given in its integrable form as

$$\begin{aligned} &\mathrm{i}q_{1Z} + \tfrac{1}{2}q_{1TT} + q_1\sum_{n=1}^{N}|q_n|^2 + \mathrm{i}\varepsilon\left[q_{1TTT} + 6q_{1T}\sum_{n=1}^{N}|q_n|^2 + 3q_1\left(\sum_{n=1}^{N}|q_n|^2\right)_T\right] = 0 \\ &\mathrm{i}q_{2Z} + \tfrac{1}{2}q_{2TT} + q_2\sum_{n=1}^{N}|q_n|^2 + \mathrm{i}\varepsilon\left[q_{2TTT} + 6q_{2T}\sum_{n=1}^{N}|q_n|^2 + 3q_2\left(\sum_{n=1}^{N}|q_n|^2\right)_T\right] = 0 \\ &\qquad\vdots \\ &\mathrm{i}q_{NZ} + \tfrac{1}{2}q_{NTT} + q_N\sum_{n=1}^{N}|q_n|^2 + \mathrm{i}\varepsilon\left[q_{NTTT} + 6q_{NT}\sum_{n=1}^{N}|q_n|^2 + 3q_N\left(\sum_{n=1}^{N}|q_n|^2\right)_T\right] = 0. \end{aligned} \tag{3}$$

To analyse equation (3), it is rather convenient to introduce variable transformations:

$$
\begin{aligned}
&E_1(t,z) = q_1(T,Z)\exp\left\{\frac{-\mathrm{i}}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\}\\
&E_2(t,z) = q_2(T,Z)\exp\left\{\frac{-\mathrm{i}}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\}\\
&\vdots\\
&E_N(t,z) = q_N(T,Z)\exp\left\{\frac{-\mathrm{i}}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\}\\
&z = Z \qquad t = T - \frac{Z}{12\varepsilon}.
\end{aligned}
\tag{4}
$$

By making use of the transformations, equation (3) is reduced to N-coupled complex modified KdV-type equations of the form

$$
\begin{aligned}
&E_{1Z} + \varepsilon\left[E_{1TTT} + 6E_{1T}\sum_{n=1}^{N}|E_n|^2 + 3E_1\left(\sum_{n=1}^{N}|E_n|^2\right)_T\right] = 0\\
&E_{2Z} + \varepsilon\left[E_{2TTT} + 6E_{2T}\sum_{n=1}^{N}|E_n|^2 + 3E_2\left(\sum_{n=1}^{N}|E_n|^2\right)_T\right] = 0\\
&\vdots\\
&E_{NZ} + \varepsilon\left[E_{NTTT} + 6E_{NT}\sum_{n=1}^{N}|E_n|^2 + 3E_N\left(\sum_{n=1}^{N}|E_n|^2\right)_T\right] = 0.
\end{aligned}
\tag{5}
$$

In order to construct the explicit Lax pair for equation (5), we generalize the 2×2 Ablowitz–Kaup–Newell–Segur (AKNS) method [15] to a $(2N+1) \times (2N+1)$ eigenvalue problem. The linear eigenvalue problem is written as

$$
\left.\begin{aligned}
\Psi_t &= U\Psi\\
\Psi_z &= V\Psi
\end{aligned}\right\} \qquad \Psi = (\,\Psi_1 \quad \Psi_2 \quad \Psi_3 \quad \ldots \quad \Psi_{2N} \quad \Psi_{2N+1}\,)^T
\tag{6}
$$

where

$$
U = \begin{pmatrix}
-\mathrm{i}\lambda & E_1 & E_1^* & \cdots & E_N & E_N^*\\
-E_1^* & \mathrm{i}\lambda & 0 & \cdots & 0 & 0\\
-E_1 & 0 & \mathrm{i}\lambda & \cdots & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
-E_N^* & 0 & 0 & \cdots & \mathrm{i}\lambda & 0\\
-E_N & 0 & 0 & \cdots & 0 & \mathrm{i}\lambda
\end{pmatrix}
\tag{7}
$$

$$
V = \begin{pmatrix}
V_{11} & V_{12} & V_{13} & \cdots & V_{1(2N)} & V_{1(2N+1)}\\
V_{21} & V_{22} & V_{23} & \cdots & V_{2(2N)} & V_{2(2N+1)}\\
V_{31} & V_{32} & V_{33} & \cdots & V_{3(2N)} & V_{3(2N+1)}\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
V_{(2N)1} & V_{(2N)2} & V_{(2N)3} & \cdots & V_{(2N)(2N)} & V_{(2N)(2N+1)}\\
V_{(2N+1)1} & V_{(2N+1)2} & V_{(2N+1)3} & \cdots & V_{(2N+1)(2N)} & V_{(2N+1)(2N+1)}
\end{pmatrix}.
\tag{8}
$$

In general, the matrix V is written as

$$
V_{ij} = \sum_{n=0}^{3} V_{ij}^{(n)}\lambda^n.
\tag{9}
$$

The integrability condition for the matrices U and V is

$$
U_z - V_t + UV - VU = 0.
\tag{10}
$$

Using the integrability condition and choosing the appropriate constants of integration, the V matrix is obtained as

$$V = \frac{8\mathrm{i}\varepsilon\lambda^3}{2N+1}\begin{pmatrix} -2N & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} + 4\varepsilon\lambda^2 \begin{pmatrix} 0 & E_1 & E_1^* & \cdots & E_N & E_N^* \\ -E_1^* & 0 & 0 & \cdots & 0 & 0 \\ -E_1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -E_N^* & 0 & 0 & \cdots & 0 & 0 \\ -E_N & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

$$+2\mathrm{i}\varepsilon\lambda \begin{pmatrix} -2A & -E_{1t} & -E_{1t}^* & \cdots & -E_{Nt} & -E_{Nt}^* \\ -E_{1t}^* & |E_1|^2 & (E_1^*)^2 & \cdots & E_1^* E_N & E_1^* E_N^* \\ -E_{1t} & E_1^2 & |E_1|^2 & \cdots & E_1 E_N & E_1 E_N^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -E_{Nt}^* & E_1 E_N^* & E_1^* E_N^* & \cdots & |E_N|^2 & (E_N^*)^2 \\ -E_{Nt} & E_1 E_N & E_1^* E_N & \cdots & E_N^2 & |E_N|^2 \end{pmatrix}$$

$$-\varepsilon \begin{pmatrix} 0 & 4AE_1 + E_{1tt} & 4AE_1^* + E_{1tt}^* & \cdots & 4AE_N + E_{Ntt} & 4AE_N^* + E_{Ntt}^* \\ -4AE_1^* - E_{1tt}^* & E_1 E_{1t}^* - E_1^* E_{1t} & 0 & \cdots & E_N E_{1t}^* - E_1^* E_{Nt} & E_N^* E_{1t}^* - E_1^* E_{Nt}^* \\ -4AE_1 - E_{1tt} & 0 & E_1^* E_{1t} - E_1 E_{1t}^* & \cdots & E_N E_{1t} - E_1 E_{Nt} & E_N^* E_{1t} - E_1 E_{Nt}^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -4AE_N^* - E_{Ntt}^* & E_1 E_{Nt}^* - E_N^* E_{1t} & E_1^* E_{Nt}^* - E_N^* E_{1t}^* & \cdots & E_N E_{Nt}^* - E_N^* E_{Nt} & 0 \\ -4AE_N - E_{Ntt} & E_1 E_{Nt} - E_N E_{1t} & E_1^* E_{Nt} - E_N E_{1t}^* & \cdots & 0 & E_N^* E_{Nt} - E_N E_{Nt}^* \end{pmatrix} \tag{11}$$

where $A = \sum_{n=1}^{N} |E_n|^2$.

Hence, the complete integrability of equation (5) and thereby equation (3) is confirmed by the Lax pair given above. From this Lax pair, the soliton solutions can be generated using various analytical methods. Here, we use the Darboux–Bäcklund transformation and the explicit soliton solutions are generated.

To derive the Bäcklund transformation of equation (5), we have to write down equation (6) in the coupled Riccati form. Introducing new variables (or pseudopotentials [16])

$$\Gamma_1 = \frac{\Psi_1}{\Psi_{2N+1}} \quad \Gamma_2 = \frac{\Psi_2}{\Psi_{2N+1}} \quad \Gamma_3 = \frac{\Psi_3}{\Psi_{2N+1}} \ldots \Gamma_{2N} = \frac{\Psi_{2N}}{\Psi_{2N+1}} \tag{12}$$

equation (5) yields

$$\Gamma_{1t} = -2\mathrm{i}\lambda\Gamma_1 + \sum_{j=1}^{N-1}(E_j\Gamma_{2j} + E_j^*\Gamma_{2j+1}) + E_N(\Gamma_{2N} + \Gamma_1^2) + E_N^* \tag{13a}$$

$$\Gamma_{jt} = \begin{cases} -\Gamma_1 E_j^* + \Gamma_1\Gamma_{2j}E_N & \text{for even } N \\ -\Gamma_1 E_j + \Gamma_1\Gamma_{2j+1}E_N & \text{for odd } N. \end{cases} \tag{13b}$$

Now, let us seek a transformation of variables $\Gamma_1 \to \Gamma_1'$, $\Gamma_2 \to \Gamma_2'$, $\Gamma_3 \to \Gamma_3', \ldots, \Gamma_{2N} \to \Gamma_{2N}'$, $\lambda \to \lambda'$, $E_1 \to E_1'$, $E_2 \to E_2'$, $E_3 \to E_3', \ldots, E_N \to E_N'$ which keeps the form of equations (13) invariant. The simplest transformation can be tried by setting $\Gamma_1' = \Gamma_1$, $\Gamma_2' = \Gamma_2$, $\Gamma_3' = \Gamma_3, \ldots, \Gamma_{2N}' = \Gamma_{2N}$, $\lambda' = \lambda^*$, looking for E_1', E_2' and E_N' in the form

$$E_j - E_j' = \frac{2\mathrm{i}(\lambda - \lambda^*)\Gamma_1^*\Gamma_{2j+1}}{1 + \sum_{n=1}^{2N}|\Gamma_n|^2} \qquad \text{where} \quad j = 1, 2, 3, \ldots, N-1 \tag{14a}$$

$$E_N - E_N' = \frac{2\mathrm{i}(\lambda - \lambda^*)\Gamma_1^*}{1 + \sum_{n=1}^{2N}|\Gamma_1|^2}. \tag{14b}$$

Equations (14) define the Bäcklund transformation for equation (5). The primed quantities correspond to N-soliton solutions and the unprimed quantities correspond to the $(N-1)$-soliton solutions. This means that, on the basis of a known solution (seed solution) to equation (5), we are able to find pseudopotentials (12) and, making use of (14), we can then find the desired potentials E_1 and E_2, i.e. the new solutions of equation (5).

For instance, the trivial solution of equation (5) $E_1 = E_2 = \cdots = E_N = 0$ corresponds to the following pseudopotentials (with $\lambda = \mathrm{i}\beta$):

$$\begin{aligned}
\Gamma_1(0) &= c_1 \exp(2\beta t - 8\varepsilon\beta^3 z)\\
\Gamma_2(0) &= c_2\\
\Gamma_3(0) &= c_3\\
&\vdots\\
\Gamma_{2N}(0) &= c_{2N}
\end{aligned} \tag{15}$$

where $c_1, c_2, c_3, \ldots, c_{2N}$ are arbitrary integration constants. Hence, we can find new solutions of equation (5) from (14) which are generated by the trivial one:

$$E_j(1) = 2\beta \frac{c_{2j+1}}{c_1^*} \operatorname{sech}(2\beta t - 8\varepsilon\beta^3 z) \qquad \text{where} \quad j = 1, 2, 3, \ldots, N-1 \tag{16a}$$

$$E_N(1) = 2\beta \frac{1}{c_1^*} \operatorname{sech}(2\beta t - 8\varepsilon\beta^3 z). \tag{16b}$$

Expressions (16) give the one-soliton solutions of equation (5). In a similar way, using $\Gamma_1(0)$, $\Gamma_2(0)$, $\Gamma_3(0), \ldots, \Gamma_{2N}(0)$, $E_1(1)$, $E_2(1)$, $E_3(1), \ldots, E_N(1)$, one can generate the N-soliton solutions of equation (5) in a recursive manner. Using equation (4), the one-soliton solution of equation (3) is found to be

$$q_j(1) = 2\beta \frac{c_{2j+1}}{c_1^*} \operatorname{sech}\left[2\beta\left(T - \frac{Z}{12\varepsilon}\right) - 8\varepsilon\beta^3 z\right] \exp\left[\frac{\mathrm{i}}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right] \tag{17a}$$

$$q_N(1) = 2\beta \frac{1}{c_1^*} \operatorname{sech}\left[2\beta\left(T - \frac{Z}{12\varepsilon}\right) - 8\varepsilon\beta^3 z\right] \exp\left[\frac{\mathrm{i}}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right] \tag{17b}$$

where $j = 1, 2, 3, \ldots, N-1$. The intensity profile of the one-soliton solution given by equation (17) is shown in figure 1. From the above soliton solution, the information about the soliton pulse, i.e. the pulse width, pulse intensity etc, can be obtained. By constructing the higher-order soliton solutions, one can also gain some idea about the interaction of solitons with the inclusion of higher-order linear and nonlinear effects. Work is in progress in this direction.

Now, for the sake of completeness, we proceed further to construct the Hirota bilinear form for equation (3). For this, we introduce the following transformation [17]:

$$q = \frac{G}{F} \tag{18}$$

where G is a complex function and F is a real function. Using the above transformation, we obtain the Hirota bilinear form as

$$(D_z - \mathrm{i}/2D_t^2 - \varepsilon D_t^3)G_j \cdot F = 0 \qquad \text{where} \quad j = 1, 2, 3, \ldots, N \tag{19}$$

$$D_t^2 F \cdot F = 2\left(\sum_{j=1}^{N} |G_j|^2\right) \tag{20}$$

$$D_t G_j \cdot G_j^* = 0 \qquad \text{where} \quad j = 1, 2, 3, \ldots, N \tag{21}$$

$$D_t G_j \cdot G_k = 0 \qquad \text{where} \quad j = 1, 2, 3, \ldots, N \quad k = 1, 2, 3, \ldots, N \quad j \neq k \tag{22}$$

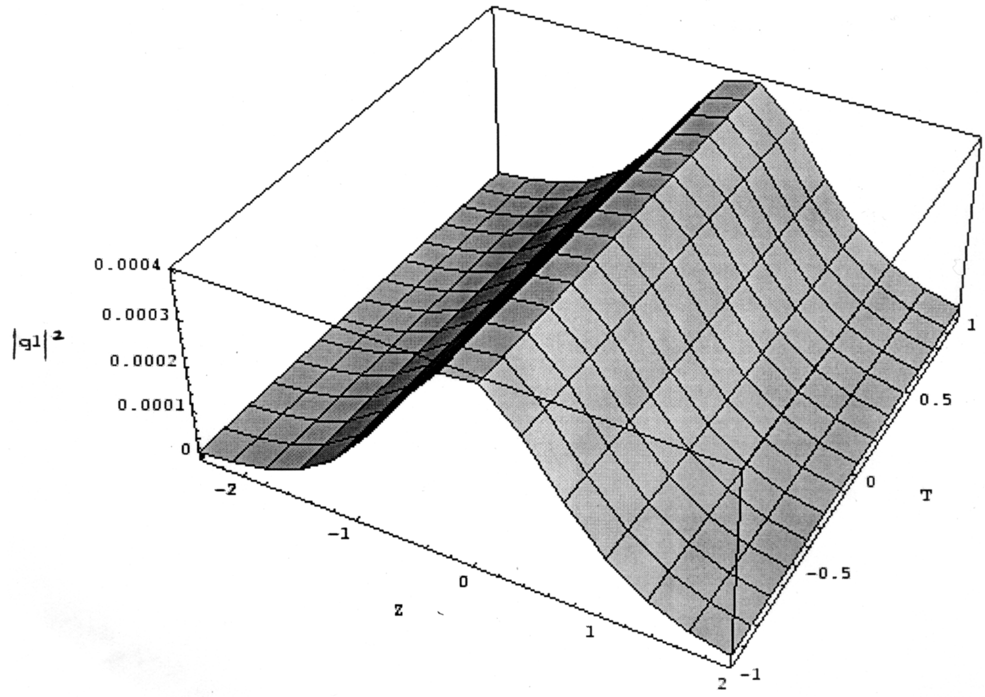

Figure 1. Intensity profile of the one-soliton solution for $\beta = 0.01$ and $\varepsilon = 0.0015$.

$$D_t G_j \cdot G_k^* = 0 \qquad \text{where} \quad j = 1, 2, 3, \ldots, N \quad k = 1, 2, 3, \ldots, N \quad j \neq k \tag{23}$$

$$D_t G_j^* \cdot G_k = 0 \qquad \text{where} \quad j = 1, 2, 3, \ldots, N \quad k = 1, 2, 3, \ldots, N \quad j \neq k. \tag{24}$$

The bilinear operator is defined by

$$D_t^m D_x^n = \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^n G(x,t)F(x',t')\bigg|_{x'=x,t'=t}. \tag{25}$$

From the bilinear form, it should be noted that equations (21)–(24) give severe constraint to the constructed form of G, F and G^*. Using the standard expansion method, one can construct the soliton solutions in a systematic manner.

Thus, in this work, we have generalized the 2×2 AKNS method to the $(2N+1)\times(2N+1)$ eigenvalue problem of N-coupled HNLS equations. We have constructed the explicit Lax pair and the exact soliton solutions using Darboux–Bäcklund transformations. We have also constructed the Hirota bilinear form for the system. Hence, with these results, we have proved that the N-coupled CHNLS equations which describe the wave propagation of N numbers of fields in a fibre system with all the higher-order effects such as TOD, Kerr dispersion and stimulated Raman effect, will allow soliton-type pulse propagation. From the soliton solutions, one can obtain the information about the shape, width and intensity of the propagating pulse.

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